Name: ______________________________

Teacher: ____________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2010

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS

(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1, 2, 3	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and logarithms	4, 5, 7, 8	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	6, 9	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

<u>Question One</u>: (Start a NEW BOOKLET)

a) Evaluate $\dfrac{\pi}{\sqrt{e^2-1}}$ correct to 3 decimal places. [2]

b) Solve $\dfrac{3-2x}{x}=4$. [2]

c) Rationalise the denominator of $\dfrac{2}{1+\sqrt{3}}$. [2]

d) Factorise $4+11x-3x^2$. [2]

e) Sketch the graph of $x+2y-6=0$, showing the intercepts on both axes. [2]

f) Find the equation (in General Form) of the line perpendicular to $4x-3y-1=0$ that passes through the point $(2,-3)$. [2]

<u>Question Two</u>: (Start a NEW BOOKLET)

a) Differentiate with respect to *x*:

i. x^2e^x [2]

ii. $(1+\tan x)^2$ [2]

b) Find

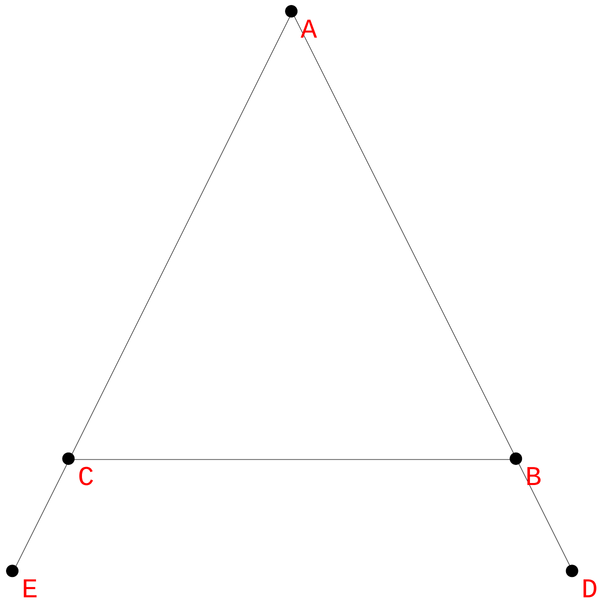
i. $\int 4x-\sin x\,dx$ [2]

ii. $\int_1^3 \dfrac{1}{x^2}\,dx$ [3]

c) Find the equation of the tangent to the curve $y=x-\dfrac{1}{x}$ at the point $(-1,0)$. [3]

<u>Question Three</u>: (Start a NEW BOOKLET)

a) Triangle ABC is isosceles with AB=AC. AB and AC are extended to D and E respectively, with BD=CE, as shown in the diagram below.



i. Copy the diagram into you answer booklet showing the information given. [1]

ii. Prove that $\Delta ABE \equiv \Delta ACD$. [4]

b) Using Simpson's Rule with five function values to find and approximate value for the integral $\int_0^2 e^{x^2}\,dx$, to 2 decimal places. [3]

c) A geometric series has a 3rd term of $\frac{1}{12}$ and an eighth term of $\frac{-1}{384}$. For this series:

i. Find an expression for T_n. [2]

ii. Find the sum of the first 8 terms. [1]

iii. Find the limiting sum. [1]

<u>Question Four</u>: (Start a NEW BOOKLET)

a) Solve $2\cos x = \sqrt{3}$ for $-\pi \le x \le \pi$. [2]

b) For the parabola $x^2 = 6\left(y+1\right)$:

i. Write down the coordinates of the vertex. [1]

ii. Find the coordinates of the focus. [1]

iii. Draw a neat sketch of the parabola. [1]

iv. Calculate the area bounded by the parabola and the line $y = 5$. [3]

c) Two ordinary dice are rolled and the score is the sum of the numbers on the top faces.

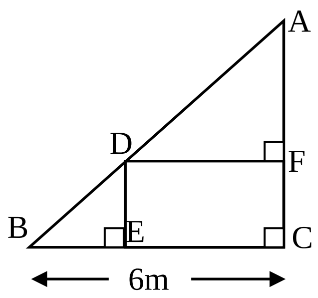
i. What is the probability that the score is 5? [2]

ii. What is the probability that the score is not 5? [1]

iii. What is the probability that the dice show "doubles" (i.e. that both numbers on the top faces are the same)? [1]

<u>Question Five</u>: (Start a NEW BOOKLET)

a) A 10m long ladder (AB) rests against a wall, with its foot (B) 6m from the base (C) of the wall (AC) as shown in the diagram below (Not drawn to Scale). D is a point on the ladder AB.



i. How far up the wall does the ladder reach? [1]

ii. Explain why DF||BC. [1]

iii. Prove $\Delta ADF \;|||\; \Delta ABC$. [2]

iv. Felix climbs the ladder to point D so that he is 3m directly above the ground (E). How far along the ladder (BD) has he climbed? [2]

b) Find the equation of the normal to the curve $y = x(x-2)$ when $x = 2$. [2]

c) Given $g(x) = ax^2 + bx + c$ and that $g(0) = 4, g(1) = 23, g(-1) = 1$, determine the values of a, b and c. [2]

<u>Question Six</u>: (Start a NEW BOOKLET)

a) For the function $y = x^3 - 3x^2 - 9x + 6$:

i. At what point does this curve cut the *y*-axis? [1]

ii. Find the coordinates of any stationary points and determine their nature. [3]

iii. Find the coordinates of any points of inflection. [1]

iv. For what values of x is the curve concave up? [1]

v. Sketch the curve, showing the information above. [2]

b) If $\sin\theta = -\dfrac{2}{3}$ and $\cos\theta > 0$, find the value of $\tan\theta$ (in surd form). [2]

c) Show that the derivative of xe^x is $e^x + xe^x$, and hence find $\int xe^x \, dx$. [2]

<u>Question Seven</u>: (Start a NEW BOOKLET)

a) If α and β are the roots of $4x^2 + 8x - 1 = 0$, find the value of

i. $\alpha + \beta$ [1]

ii. $\alpha\beta$ [1]

iii. $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ [2]

b) For the curve $y = 3\sin 2x$:

i. State the amplitude. [1]

ii. Find the period. [1]

iii. Draw a neat sketch of the curve for $0 \le x \le \pi$. [2]

c) Find the value of:

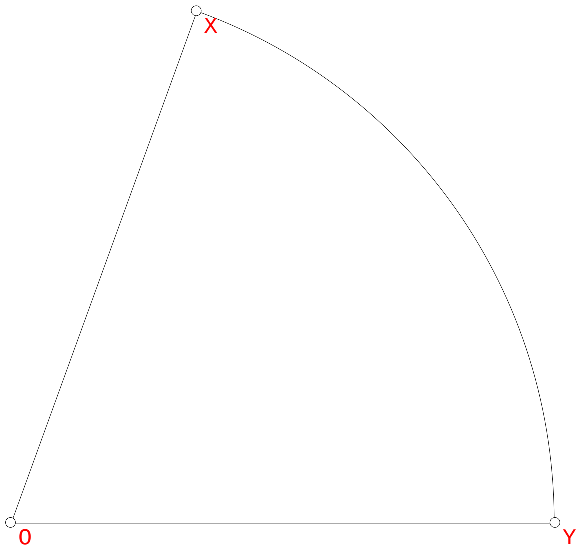
i. $\log_2 45$, given that $\log_2 3 = 1.585$ and $\log_2 5 = 2.322$, without using the change of base law. [1]

ii. $\log_7 0.3$, using change of base law. [1]

d) Find p so that $9x^2 - 3x + p = 0$ has only one root. [2]

<u>Question Eight</u>: (Start a NEW BOOKLET)

a) In the diagram, XY is an arc of a circle with centre O and radius 12cm. The length of the arc XY is 4π cm.

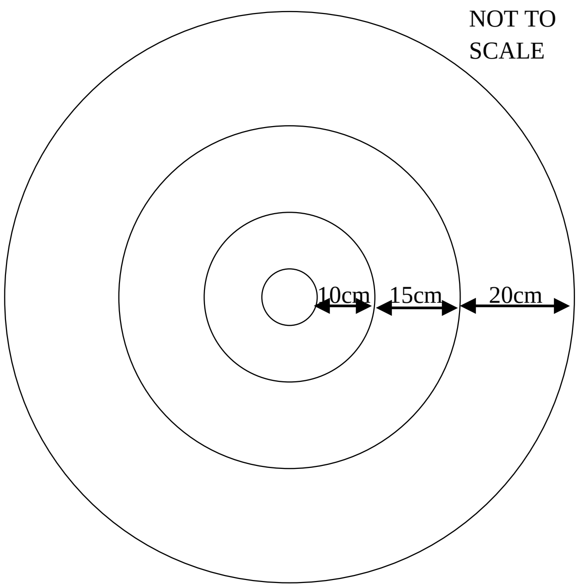


i. Find the exact size of θ in radians. [1]

ii. Find the area of the sector OXY [1]

b) The region bounded by the curve $y = e^{x} + e^{-x}$, the x-axis and the lines $x = 0$ and $x = 2$ is rotated about the x-axis. Find the volume of the solid formed. (Answer in terms of e). [3]

c) Beginning with a circular piece of fabric of radius 5cm, Lynn sewed together circular strips of different coloured fabrics which increased in width to make a circular tablecloth. The finished width of the first strip was 10cm, the second was 15cm, the third 20cm, and so on, as shown opposite.



i. Show that the width of the 10^{th} strip was 55cm. [2]

ii. The radius of the Tablecloth was 455cm. How many strips were sewn onto the edge of the first circular piece? [3]

d) Solve for x: $4e^{2x} - e^{x} = 0$. [2]

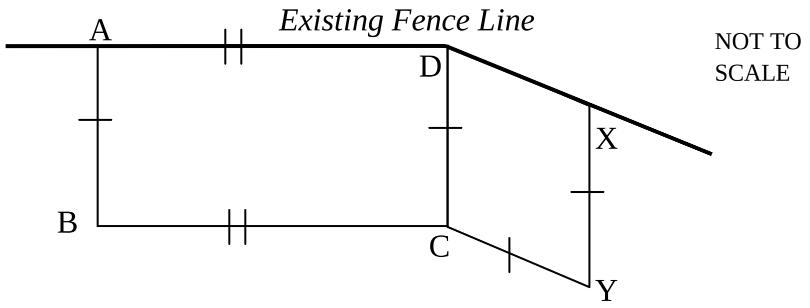
<u>Question Nine</u>: (Start a NEW BOOKLET)

a) For the inequality $y \le 4 - x^2$:

i. Shade the region bounded simultaneously by the inequality above, and the inequalities $x \ge 0$ and $y \ge 3x$. [2]

ii. Find the volume of the solid of revolution formed when the region defined in (i) above is rotated about the *y*-axis. [4]

b) A farmer needs to construct two holding paddocks, one rectangular (ABCD) and one a rhombus (CDXY) for horses and cattle respectively. The diagram below shows an aerial view of the paddocks, including the use of an existing fence as part of the boundary.



The farmer has only 700m of fencing. We also know that $\angle CDX = 30^\circ$.

i. By letting $AB = x$, show that the area A of the two paddocks is given by $A = 700x - \dfrac{7x^2}{2}$. [2]

ii. Hence find the maximum area that can be enclosed. [3]

iii. Calculate the dimensions for the rectangular paddock, when the overall area is a maximum. [1]

<u>Question Ten</u>: (Start a NEW BOOKLET)

a) Jerry joins a Superannuation fund, investing $P at the beginning of every year at 9% p.a. compounding annually.

i. Write an expression for the value of his investment A_1 at the end of the first year. [1]

ii. Write an expression for the value of his investment A_2 at the end of the second year. [1]

iii. Show that, after n years, the value of his investment A_n is given by

$$A_n = \frac{109P}{9}\left(1.09^n - 1\right)$$ [2]

iv. If, after 30 year, he wishes to collect $1,000,000, calculate the value of $P to the nearest dollar. [1]

b) A triangle is right-angled at *B*. *D* is the point on *AC* such that *BD* is perpendicular to *AC*. Let $\angle BAC = \theta$.

i. Draw a diagram showing this information. [1]

You are given that $6AD + BC = 5AC$:

ii. Show that $6\cos\theta + \tan\theta = 5\sec\theta$ [2]

iii. Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$ [2]

iv. Find θ. [2]

Question One: — **Marking** — **Comments**

a) 1.242886646 — ❶ answer

=1.243 (to 3dp) — ❶ dp’s

b) $3-2x=4x$ — ❶ algebra

$$3=6x$$

$$x=\frac{1}{2}$$ — ❶ answer

c) $\frac{2}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}$ — ❶ $\times$conjugate

$$=\frac{2(1-\sqrt{3})}{1-3}$$

$$=\frac{2(1-\sqrt{3})}{-2}$$

$=\sqrt{3}-1$ — ❶ answer — In simplest form!

d) $=4+12x-x-3x^2$ — ❶ resolves pairs

$=4(1+3x)-x(1+3x)$

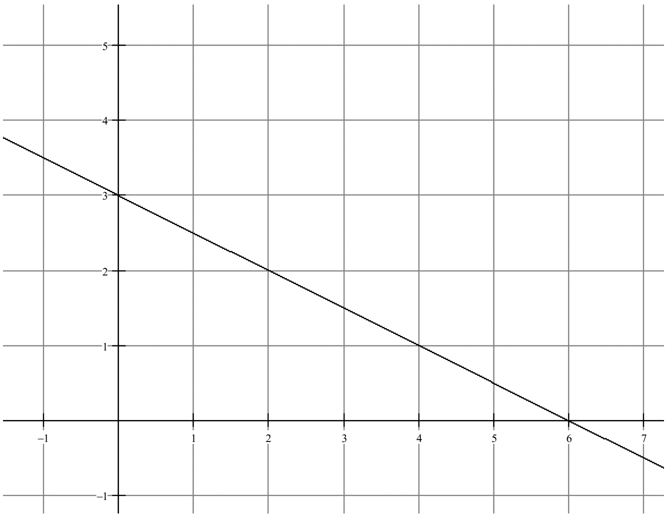
$=(1+3x)(4-x)$ — ❶ answer

e) $2y=6-x$

$$y=3-\frac{1}{2}x$$

f) $x=0:$ $y=0:$ — ❶ intercepts — Some had trouble finding these!

$y=3$ $x=6$



❶graph

g) $3y=4x-1$

$y=\frac{4}{3}x-\frac{1}{3}\Rightarrow m_1=\frac{4}{3}$, so perp. gives $m_2=-\frac{3}{4}$ — ❶ perp. gradient

So through $(2,-3)$:

$$y-(-3)=-\frac{3}{4}(x-2)$$

$$4y+12=-3x+6$$

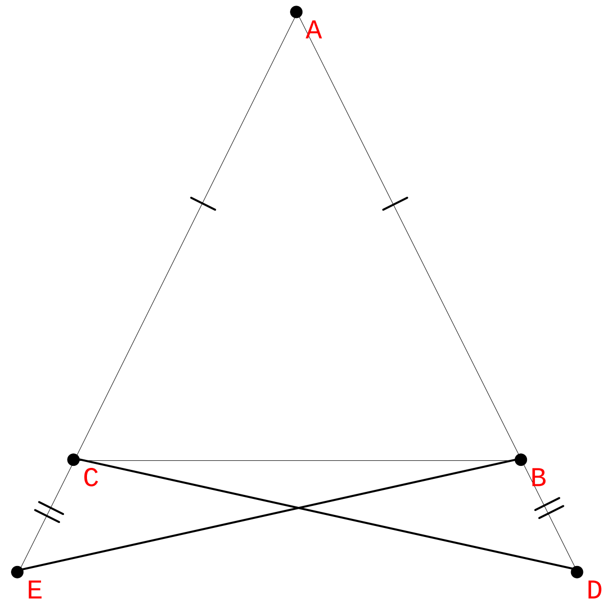
$3x+4y+6=0$ — ❶ eqn (in GF) — Again, simplify!

Question Two:

	Marking	Comments
a) i) $u = x^2 \quad v = e^x$ $du = 2x \quad dv = e^x$ $\therefore \dfrac{d(x^2 e^x)}{dx}$ $= x^2 e^x + 2xe^x$	❶ product rule	Mostly good
$= xe^x(x+2)$	❶ answer	
ii) $\dfrac{d(1+\tan x)^2}{dx}$ $= 2(1+\tan x)^1 \times \dfrac{d(\tan x)}{dx}$	❶chain rule	Some struggled with structure of the Chain Rule.
$= 2\sec^2 x(1+\tan x)$	❶ answer	Some did not know the derivative of tan*x*.
b) i) $\int 4x - \sin x \, dx$	❶ answer	Quite a few integration errors, and some forgot "+c"
$= 2x^2 + \cos x + c$	❶ for '+*c*'	
ii) $\int_1^3 \dfrac{1}{x^2} dx$ $= \left[-\dfrac{1}{x}\right]_1^3$	❶ int & limits	Many 2 Unit candidates got a log or differentiated.
$= \dfrac{-1}{3} - \dfrac{-1}{1}$	❶ subst	Many also made sign errors in substitution.
$= \dfrac{2}{3}$	❶ answer	
c) $\dfrac{dy}{dx} = 1 + \dfrac{1}{x^2}$,	❶ derivative	Some could not find derivative.
so when $x = -1$ $\dfrac{dy}{dx} = 2$	❶ gradient	Many errors to find "m=2" ("m=0" was popular).
Hence $y - 0 = 2(x+1)$ or $y = 2x + 2$	❶ answer	More seriously, not finding an "m" value but using algebra: $y = \left(1 + \dfrac{1}{x^2}\right)(x+1)$ is meaningless!

Question Three: **Marking** **Comments**

a) i)



❶ markings

Some students failed to give all the information required.

ii) AB=AC and BE=CD (given)

AD=AB+BD, AE=AC+CE (st. lns), so

AD=AC+CE, hence

AD=AE ❶ reason

Some did not show this (or equivalent);

In $\Delta's\ ABE, ACD$ ← Need to state the triangles;

i) AB = AC (given) ❶ reason

ii) $\angle EAB = \angle DAC$ (common angle) ❶ reason

iii) AD=AE (shown above)

$\therefore \Delta ABE \equiv \Delta ACE$ (SAS) ❶ conclusion

Some did not state the test used.

b) For $y = e^{x^2}$, with $h = 0.5$: ❶ values

x	0	0.5	1	1.5	2
y_i	1	$e^{0.25}$	e	$e^{2.25}$	e^4

Simpsons Rule:

$$A = \frac{h}{3}\left[\left(y_0 + y_4\right) + 2\left(y_1 + y_3\right) + 4y_2\right]$$ ❶ subst

$$= \frac{0.5}{3}\left[\left(1 + e^4\right) + 4\left(e + e^{2.25}\right) + 2e\right]$$

$\doteq 17.35362645$

$\doteq 17.35$ ❶ ans to 2dp

Some students had the 4 and the 2 the wrong way around in the formula.

c) With $T_3 = \frac{1}{12}, T_8 = \frac{-1}{384}$:

i. $ar^2 = \frac{1}{12}$ and $ar^7 = \frac{-1}{384}$, hence

$$\frac{T_8}{T_3} = \frac{-1}{384} \div \frac{1}{12}$$

$$\frac{ar^7}{ar^2} = \frac{-1}{384} \times \frac{12}{1}$$

$$r^5 = \frac{-1}{32}$$

$$r = \frac{-1}{2}$$

This part done well provided the correct ratio of $\frac{-1}{2}$ found (some used $\frac{\pm 1}{2}$ or $\frac{1}{2}$.

$$a.\left(\frac{-1}{2}\right)^2 = \frac{1}{12}$$

❶ a, r values

$$a = \frac{1}{3}$$

$$T_n = ar^n$$

$$= \frac{1}{3}.\left(\frac{-1}{2}\right)^n$$

❶ T_n correct

ii. $$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$S_8 = \frac{\frac{1}{3}\left(1 - \left(\frac{-1}{2}\right)^8\right)}{1 - \frac{-1}{2}}$$

$$= \frac{\frac{1}{3}\left(1 - \frac{1}{256}\right)}{\frac{3}{2}}$$

$$= \frac{1}{3}.\frac{2}{3}.\frac{255}{256}$$

$$= \frac{85}{384}$$

❶ S_8 correct

This answer best left as a fraction.

iii. $$S_\infty = \frac{a}{1 - r}$$

$$= \frac{\frac{1}{3}}{1 - \frac{-1}{2}}$$

$$= \frac{1}{3}.\frac{2}{3}$$

$$= \frac{2}{9}$$

❶ S_∞ correct

<u>Question Four</u>: — **Marking** — **Comments**

a) $\cos x = \dfrac{\sqrt{3}}{2}$

$x = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{\pi}{6}$ — ❶ base angle

$\cos x$ positive in Q1 & Q4, but with $-\pi \le x \le \pi$,

$x = \dfrac{\pi}{6}, \dfrac{-\pi}{6}$ — ❶ answers

Comments: Many did not read the requirements of the question: $-\pi \le x \le \pi$ also implies radians!

b)

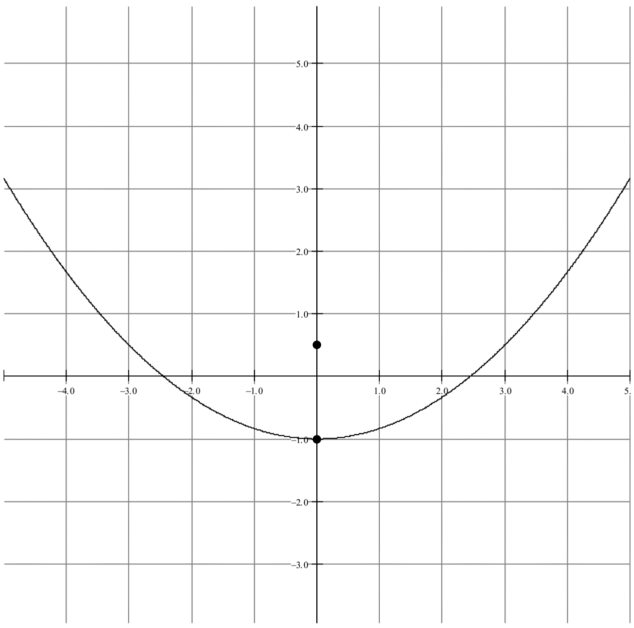
i. Vertex is $(0,-1)$ — ❶ answer

ii. $4a = 6$

Hence $a = \dfrac{3}{2}$, so focus is $(0,-1+a)$ or

$\left(0,\dfrac{1}{2}\right)$ — ❶ answer

iii.



❶ graph

Comments: Sketch was poorly done by many – must show vertex, focus, intercepts.

Use a ruler to draw axes!

iv. $y = 5$ and $x^2 = 6(y+1)$ — ❶ eqn for Int

$x^2 = 6(5+1)$

$= 36$

$y + 1 = \dfrac{x^2}{6}$

$y = \dfrac{x^2}{6} - 1$

$x = \pm 6$, hence — ❶ boundaries

Comments: Poorly done. Students need to review area between two curves.

$$A = \int_{-6}^{6} 5 - \left(\frac{x^2}{6} - 1\right) dx$$

$$= 2\int_{0}^{6} 5 - \left(\frac{x^2}{6} - 1\right) dx$$

$$= 2\left[6x - \frac{x^3}{18}\right]_0^6$$

$$= 2\left[\left(6^2 - \frac{6^3}{18}\right) - 0\right]$$

$$= 48$$ ❶ answer

c)

i. First die 1 to 4 must correspond to 2nd die 4 to1, so 4 outcomes give a total of 5 ❶ reason for 4

$$P(5) = \frac{4}{36}$$

$$= \frac{1}{9}$$ ❶ answer

ii. $P(not\,5) = 1 - P(5)$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9}$$ ❶ answer

iii. $P(doubles) = \frac{1}{6}$ ❶ answer

Question Five:	**Marking**	**Comments**
a)		
i. By Pythagoras: $AC^2 = 10^2 - 6^2$ $= 64$		
$AC = 8$	❶ answer	
ii. $\angle DFA = \angle BCA = 90^\circ$ For BC and DF, $\angle DFA$ and $\angle BCA$ are in a corresponding position and equal. Hence $BC \parallel DF$.	❶ reasoning	Some students failed to recognise corresponding angles, or failed to write that fact down!
iii. In $\Delta's\ ADF, ABC$ $\angle DFA = \angle BCA = 90^\circ$ (given in diagram) $\angle A$ is common.	❶ reasons	
Hence all angles are equal so $\Delta AFD \parallel\!\mid \Delta ABC$	❶ conclusion	
iv. Similarly, $\Delta BDE \parallel\!\mid \Delta ABC$, hence $\frac{DE}{AC} = \frac{DB}{AB}$		
$\frac{3}{8} = \frac{DB}{10}$	❶ subst	
$DB = 3.75$	❶ answer	
b) $y = x^2 - 2x$; $x = 2, y = 0$ $\frac{dy}{dx} = 2x - 2; x = 2, \frac{dy}{dx} = 2$, so normal gradient is		Parts b) and c) generally well done.
$\frac{-1}{2}$	❶ gradient	
$y - 0 = \frac{-1}{2}(x - 2)$ $y = 1 - \frac{x}{2}$		
$0 = x + 2y - 2$	❶ answer	
c) $g(0) = 4 : 4 = a.0^2 + b.0 + c$ $g(1) = 23 : 23 = a.1^2 + b.1 + c$ $g(-1) = 1 : 1 = a.(-1)^2 + b.(-1) + c$, giving the eqns $4 = c \quad \langle 1 \rangle$		
$23 = a + b + c \quad \langle 2 \rangle$	❶ set-up	
$1 = a - b + c \quad \langle 3 \rangle$ $\langle 1 \rangle$ in $\langle 2 \rangle$ and $\langle 3 \rangle$ gives: $a + b = 19$ $a - b = -3$, then adding gives: $2a = 16$; back-substitution gives $23 = 8 + b + 4$ $a = 8$ $\quad$ $b = 11$		
Hence $a = 8, b = 11, c = 4$	❶ answers	

<u>Question Six</u>: | **Marking** | **Comments**

a)

i. y-intercept at $(0,6)$. — ❶ answer

ii. $y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

Stat Pts when $y' = 0$:

$0 = 3x^2 - 6x - 9$

$= x^2 - 2x - 3$

$= (x-3)(x+1)$

$\therefore x = -1, 3$ — ❶ x values

$x = -1$ $\qquad$ $x = 3$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 6$ $\qquad$ $y = (3)^3 - 3(3)^2 - 9(3) + 6$

$= 11$ $\qquad$ $= -21$

Pts are $(-1, 11)$ and $(3, -21)$

$x = -1 \qquad x = 3$

$y'' = -12 \qquad y'' = 12$

$\Rightarrow ccd \qquad \Rightarrow ccu$ — ❶ test

$\therefore$ $(-1, 11)$ is a max t.p.

and $(3, -21)$ is a min t.p. — ❶ points

Comments: Make sure co-ordinates are stated when the question asks for them

iii. $y'' = 0 : 0 = 6x - 6$, hence possible inflection pt when $x = 1$.

when $x < 1, y'' < 0 \Rightarrow ccd$

when $x > 1, y'' > 0 \Rightarrow ccu$

hence concavity changes, so $(1, -5)$ is a point of inflexion. — ❶point & test

Comments: <u>Must</u> test the nature of Stat. Pts and Inflexion Pts!

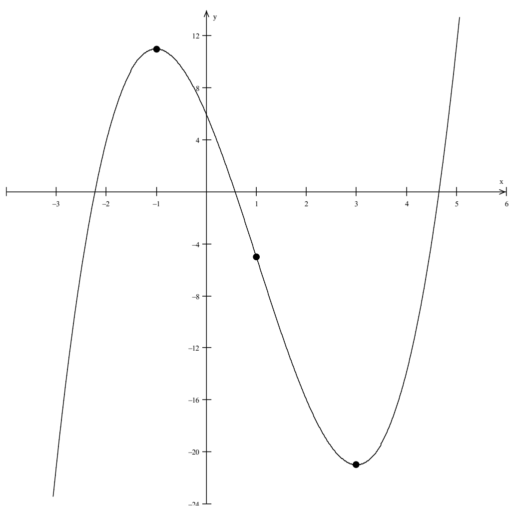
iv. Concave up when $y'' > 0$

$6x - 6 > 0$

$6x > 6$

i.e. when $x > 1$ — ❶ answer

v.



❶ t.p's

❶ intercepts

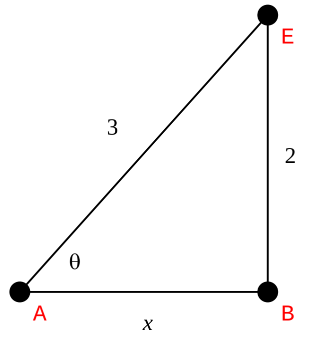
Comments: Sketch very poorly done.
Use:

- a ruler for axes
- a suitable scale

Show the information requested:

- turning points
- inflexion point
- intercepts (x and y)

b) $\sin\theta = \frac{-2}{3}$



Some students did not know the ASTC results.

Hence $x^2 = 3^2 - 2^2$, or $x = \sqrt{5}$

$\sin\theta < 0 \Rightarrow Q_3, Q_4$

$\cos\theta > 0 \Rightarrow Q_1, Q_4 \Rightarrow \tan\theta \, in \, Q_4; \tan\theta < 0$ ❶ quadrant

Hence $\tan\theta = \frac{-2}{\sqrt{5}}$ ❶ answer

c) $\frac{d(xe^x)}{dx}$ $\quad u = x \quad v = e^x$; $\quad u' = 1 \quad v' = e^x$ ❶ product rule

Showing a result – you must clearly demonstrate the link for each step.

$= xe^x + 1.e^x$

$= e^x + xe^x$ as reqd.

Hence $\frac{d(xe^x)}{dx} = e^x + xe^x$, so integrating gives:

$$xe^x = \int (e^x + xe^x)\,dx$$

$$= \int e^x\,dx + \int xe^x\,dx$$

$$\therefore \int xe^x\,dx = xe^x - \int e^x\,dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1) + c$$ ❶ answer

Question Seven: **Marking** **Comments**

a) $4x^2+8x-1=0$:

i. $\alpha+\beta=\frac{-8}{4}$

$=-2$ — ❶ answer

ii. $\alpha\beta=\frac{-1}{4}$ — ❶ answer

iii. $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ — ❶ resolves — Many had problems with this identity

$=\frac{(-2)^2-2.\frac{-1}{4}}{\left(\frac{-1}{4}\right)^2}$

$=16\left(4+\frac{1}{2}\right)$

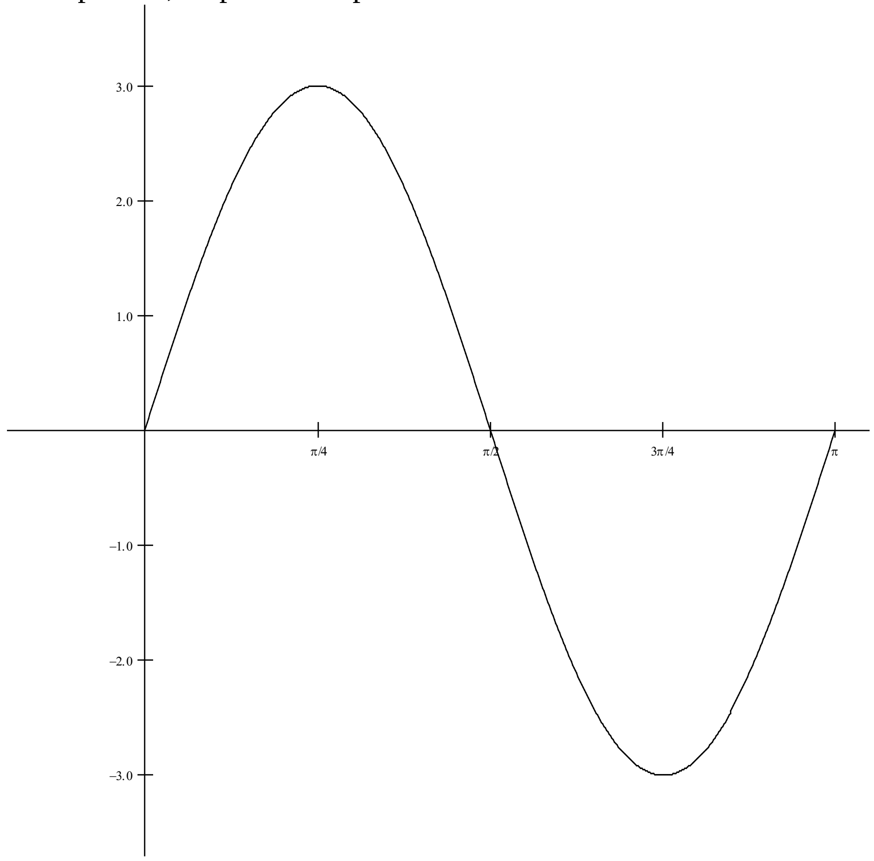
$=72$ — ❶ answer

b) $y=3\sin 2x$

i. Amplitude is 3 — ❶ answer

ii. Period is $\frac{2\pi}{2}$, or π — ❶ answer

iii. ❶ amplitude, ❶ period/shape — Some drew Cos!



c)

i. $\log_2 45$

$= \log_2 (5 \times 9)$

$= \log_2 5 + \log_2 3^2$

$= \log_2 5 + 2\log_2 3$

$= 2.322 + 2 \times 1.585$

$= 5.492$ ❶ answer

Many multiplied the logs, instead of adding them.

ii. $\log_7 0.3$

$= \dfrac{\ln 0.3}{\ln 7}$

$= -0.6187196284$

$\simeq -0.619$ ❶ answer

d) $9x^2 - 3x + p = 0$

For only one root, $\Delta = 0$

Too many had $\Delta > 0$ for one real root!

$\therefore b^2 - 4ac = 0$

$0 = 9 - 4.9.p$ ❶ set-up

$36p = 9$

$p = \dfrac{1}{4}$ ❶ answer

This was also successfully resolved by sums and products of roots by many:

Alternate marking for (d):

$\alpha + \beta = -\dfrac{-3}{9}$, $\alpha\beta = \dfrac{p}{9}$ $\alpha = \beta$, giving:

$2\alpha = \dfrac{1}{3}$

$\alpha = \dfrac{1}{6} (= \beta)$ ❶ uses = roots

$\left(\dfrac{1}{6}\right)^2 = \dfrac{p}{9}$

$p = \dfrac{9}{36}$

$= \dfrac{1}{4}$ ❶ answer

Question Eight:

Marking | **Comments**

a)

i. $l = r\theta$

$4\pi = 12\theta$

$\therefore \theta = \dfrac{\pi}{3}$ ❶ answer

Comments: Well done.

ii. $A = \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2}.12^2.\dfrac{\pi}{3}$

$= 24\pi \ cm^2$ ❶ answer

Comments: Mostly good. A few tried to find the area of a segment or triangle.

b) $y = e^x + e^{-x}$, so

$y^2 = (e^x + e^{-x})^2$

$= e^{2x} + 2e^x e^{-x} + e^{-2x}$

$= e^{2x} + e^{-2x} + 2$ ❶ y^2 correct

Volume is given by:

$$v = \int_0^2 \pi y^2 \, dx$$

$$= \pi \int_0^2 e^{2x} + e^{-2x} + 2 \, dx$$

$$= \pi \left[\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x \right]_0^2$$ ❶ int & limits

$$= \pi \left[\left(\frac{1}{2}e^4 - \frac{1}{2}e^{-4} + 4 \right) - \left(\frac{1}{2}e^0 - \frac{1}{2}e^0 + 0 \right) \right]$$

$$= \frac{\pi}{2}\left(e^4 - e^{-4} + 8\right) cu.units$$ ❶ answer

Comments: Students who could expand y^2 correctly generally got full marks.
A few made horrible integration attempts ☹: $\int e^{2x}dx = \left[\dfrac{e^{x^2}}{x^2}\right]$
Some mistakes substituting: $e^0 - e^0$

c)

i. $T_1 = 10, T_2 = 15, T_3 = 20...$

$T_2 - T_1 = 5; T_3 - T_2 = 5$, so this is an AP with $a = 10, d = 5$, hence

$T_n = 10 + 5(n-1)$

$= 5 + 5n$ ❶ justifies AP then shows subst

When $n = 10$

$T_n = 5 + 5.10$

$= 55$ ❶ to get answer

ii. $S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$, so radius will be

$r = 5 + S_n$, so ❶ radius correct

$S_n = 455 - 5$

$= 450$

Comments: i) Was poorly set out – when a question says "Show that..." you must show yout thinking, theory used and calculations!
For full marks, students needed to <u>show</u> that they recognised an AP and then substituted.
A good answer would have <u>proved</u> an AP (as shown in solns).

$$450 = \frac{n}{2}\left(2\times 10 + 5(n-1)\right)$$
$$900 = n(15+5n)$$
$$0 = 5n^2 + 15n - 900$$
$$= n^2 + 3n - 180$$
$$0 = (n+15)(n-12)$$
$$n = -15, 12$$

As $n > 0$, there are 12 strips needed.

❶ forms quadratic

❶ answer justified

Students needed to recognise the extra 5cm centre and deduct it; for full marks, justification for the positive solution was needed.

d) $4e^{2x} - e^x = 0$, let $u = e^x$, then

$$0 = 4u^2 - u$$
$$= u(4u-1)$$

$u = 0, \frac{1}{4}$, hence

❶ resolves quad

$$e^x = 0, e^x = \frac{1}{4}$$

$e^x = 0$ has no solution.

$$e^x = \frac{1}{4}$$
$$x = \ln\left(\frac{1}{4}\right)$$
$$= -2\ln 2$$
$$(= -1.386294361)$$

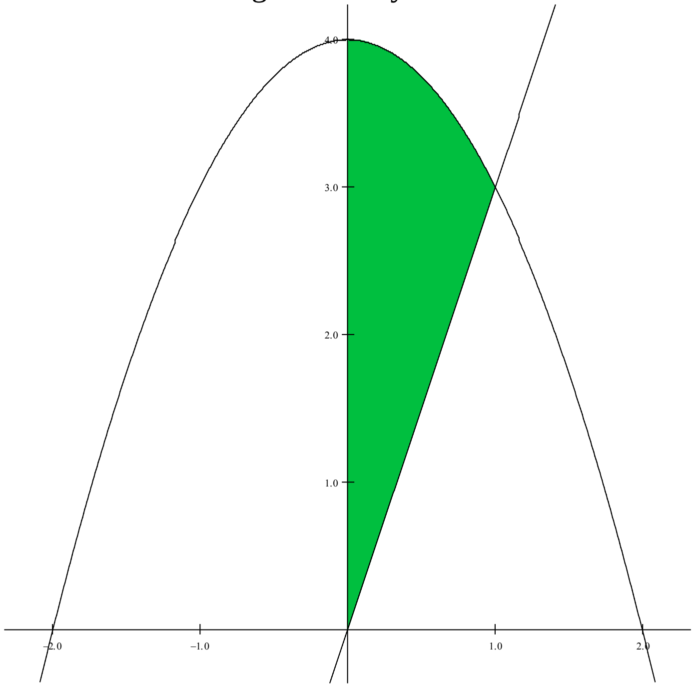
❶ answer justified

For full marks, students needed to give the two expressions for e^x and state that $e^x = 0$ has no solution.

Question Nine: **Marking** **Comments**

a) $y \le 4 - x^2$

i. ❶ shading below parabola,
❶ shading between y-axis and line.



ii. Solving $y = 3x$ and $y = 4 - x^2$ simultaneously:

$$3x = 4 - x^2$$
$$0 = x^2 + 3x - 4$$
$$= (x+4)(x-1)$$
$$x = -4, 1$$

Hence intersection at $(1, 3)$ ❶ values

So volume of solid to y-axis is $V = \pi \int_a^b x^2 \, dy$

$y = 4 - x^2$ becomes $x^2 = 4 - y$

$y = 3x$ becomes $x = \frac{y}{3}$ or $x^2 = \frac{y^2}{9}$ ❶ both x^2 eqns

$$V = \pi \int_0^3 \frac{y^2}{9} \, dy + \pi \int_3^4 4 - y \, dy$$ ❶ int & limits

Comments: Poorly done. Most students failed to establish the equation in terms of *y*, and hence did not find the correct *y* values for the integration. Many also did not recognise the need to split the integral into two parts.

$$= \pi \int_0^3 \frac{y^2}{9}\, dy + \int_3^4 4 - y\, dy$$

$$= \pi \left(\left[\frac{y^3}{27} \right]_0^3 + \left[4y - \frac{y^2}{2} \right]_3^4 \right)$$

$$= \pi \left(\left(\frac{27}{27} - 0 \right) + \left(\left(16 - \frac{16}{2} \right) - \left(12 - \frac{9}{2} \right) \right) \right)$$

$$= \pi \left(1 + 8 - \frac{15}{2} \right)$$

$$= \frac{3\pi}{2}\, cu.units$$ ❶ answer

b) $AB = x$, hence CD, CY and XY are all also x.

i. Total length of fencing is given by:

$700 = AB + CD + CY + XY + BC$

$BC = 700 - 4x$ ❶ Perim link

Poorly done. Most students did not use the sine version of the area of a triangle.

For rhombus *CDXY*:

$$A = 2 \times \frac{1}{2} CD.DY.\sin 30^\circ$$

$$= \frac{x^2}{2}$$

For rectangle *ABCD*:

$$A = AB.BC$$

$$= x(700 - 4x)$$

$$= 700x - 4x^2$$

Total area is therefore: ❶ areas & alg

$$A = 700x - 4x^2 + \frac{1}{2}x^2$$

$$= 700x - \frac{7x^2}{2}$$ as reqd.

ii. For a possible maximum, $\frac{dA}{dx} = 0$:

$\therefore \frac{dA}{dx} = 700 - 7x$; $\frac{dA}{dx} = 0$ gives

$0 = 700 - 7x$

$x = 100$ ❶ *x*-value

$\frac{d^2A}{dx^2} = -7 \Rightarrow ccd$, or a max tp. ❶ max shown

Hence max area is

$$A = 700 \times 100 - \frac{7 \times 100^2}{2}$$

$$= 35000\ sq.m$$ ❶ answer

iii. Paddock is therefore $100m$ by $300m$. ❶ answer

Question Ten: **Marking** **Comments**

a) $P invested at 9% p.a.

i. First Year: $A_1 = P(1+0.09)$

or $A_1 = 1.09P$ — ❶ answer

ii. Second Year: $A_2 = (A_1 + P)(1+0.09)$

or $A_2 = (1.09P + P)(1.09)$

$= 1.09^2 P + 1.09P$

$= P(1.09^2 + 1.09)$ — ❶ answer

iii. After n years: $A_n = \left(A_{n-1} + P\right)(1+0.09)$

Using the above pattern, this becomes:

$A_n = P(1.09^n + 1.09^{n-1} + ... + 1.09)$

Now, this is a GP with

$a = 1.09, r = 1.09, n = n$, thus — ❶ GP & values

Comment: Need to state this is a GP and write the formula

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$= \frac{1.09(1.09^n - 1)}{1.09 - 1}$$

$$= \frac{1.09(1.09^n - 1)}{0.09}$$

$$= \frac{1.09}{0.09}(1.09^n - 1)$$

$$= \frac{109(1.09^n - 1)}{9}$$

❶ GP resolved

Comment: This step 'fudged' by many.

Hence $A_n = \dfrac{109P}{9}(1.09^n - 1)$, as reqd.

iv. For A_n =$1,000,000 and n=30:

$$1,000,000 = \frac{109P}{9}(1.09^{30} - 1)$$

$$\frac{9000000}{109} = P(1.09^{30} - 1)$$

$$P = \frac{9000000}{109(1.09^{30} - 1)}$$

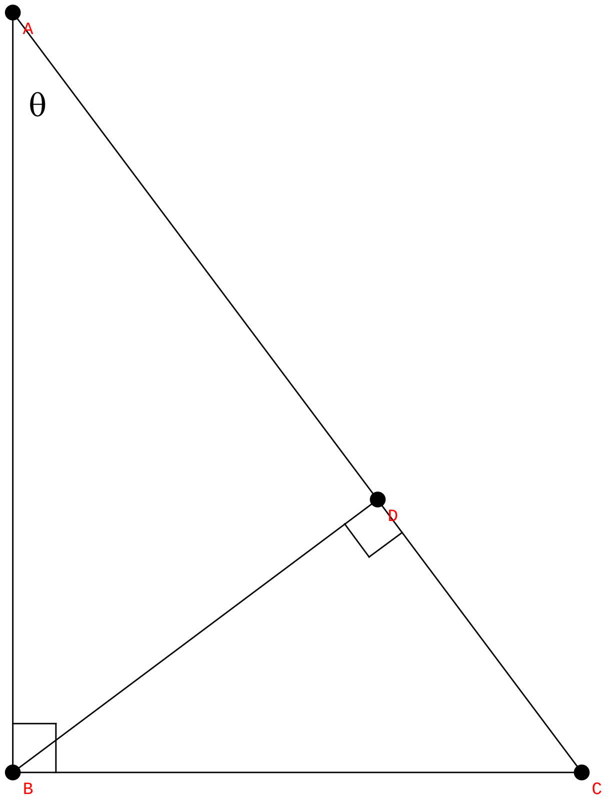
$= 6730.597606$

$= \$6731$ (nearest $) — ❶ answer

Comment: Check degree of accuracy required (no marks lost if gave to nearest ¢.

b)

i. ❶ diagram



ii. Given $6AD + BC = 5AC$, expressions for AD, BC and AC in terms of θ:

If students saw the common connection of AB, they generally did well with the question.

$\cos\theta = \dfrac{AD}{AB}$ (from ΔADB)

$AD = AB\cos\theta$

$\tan\theta = \dfrac{BC}{AB}$ and $\cos\theta = \dfrac{AB}{AC}$ (from ΔABC)

$BC = AB\tan\theta$ $AC = \dfrac{AB}{\cos\theta}$

❶ exp for each

Substituting these into the expression above:

$$6AB\cos\theta + AB\tan\theta = 5\frac{AB}{\cos\theta}$$

❶ correct subst & alg

Hence, $6\cos\theta + \tan\theta = 5\sec\theta$ as reqd.

iii. $6\cos\theta + \tan\theta = 5\sec\theta$, becomes

Many poor with Trig identities!

$$6\cos\theta + \frac{\sin\theta}{\cos\theta} = \frac{5}{\cos\theta}$$

$$6\cos^2\theta + \sin\theta = 5$$

❶ sin/cos resolved

$$6(1-\sin^2\theta) + \sin\theta = 5$$

$$6 - 6\sin^2\theta + \sin\theta - 5 = 0$$

❶ subst & alg

Hence $6\sin^2\theta - \sin\theta - 1 = 0$ as reqd.

iv.	Let $u = \sin\theta$ $0 = 6u^2 - u - 1$ $= 6u^2 - 3u + 2u - 1$ $= 3u(2u-1) + 1(2u-1)$ $= (2u-1)(3u+1)$	❶ quad resolved	Need to solve 2 eqns (a negative angle was not acceptable – need the reflex angle.
	Hence $0 = 2\sin\theta - 1$ $\quad$ $0 = 3\sin\theta + 1$ $\frac{1}{2} = \sin\theta$ $\quad$ $\frac{-1}{3} = \sin\theta$ $\theta = 30°$ *or* $\theta \simeq 199°28'$ Reject $199°28'$, as θ is in a right triangle, so $\theta = 30°$	❶ θ correct	Need to give both solutions and reject the invalid one with correct reasoning given.